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C-field Cosmological Models with Variable *G* in FRW Space–Time

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Abstract C-field cosmological models based on Hoyle-Narlikar theory with variable gravitational constant G in the frame work of FRW (Friedmann-Robertson-Walker) space–time for positive and negative curvatures are investigated. To get the deterministic solutions in terms of cosmic time t, we have assumed $G = R^n$ and discussed for $n = -1, -2$, R being scalar factor. In both the cases, creation field C increases with time, the gravitational constant G and matter density (ρ) decrease with time in the model (21). In the model (41) G decreases with time and matter density (ρ) is constant. The other physical aspects of the models are also discussed.

Keywords C-field cosmology · Variable G

In Einstein's General Theory of Relativity, the gravitational constant G plays the role of coupling constant between geometry and matter. Dirac [1] proposed a theory with variable gravitational constant G motivated by large numbers hypothesis. Subsequently, many scalar tensor theories of gravity were developed in which Einstein's general theory of relativity was generalized by including a variable G satisfying conservation equation. Jordan [2] formulated the first scalar tensor theory of gravity. The extensions of Einstein's general theory of relativity with time dependent G have also been proposed by Brans and Dicke [3], Hoyle and Narlikar [4, 5] and Canuto et al. [6] to achieve possible unification of gravitation and elementary particle physics or to incorporate Mach's principle in general relativity. Smoot et al. [7] pointed out that the predictions of the Friedmann-Robertson-Walker type models do not always meet our expectations as per astronomical observations. Thus alternative theories were proposed from time to time. Hoyle and Narlikar [8] adopted a field theoretic approach introducing massless and chargeless scalar field in the Einstein-Hilbert action to account for

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the creation of matter. In the C-field (creation-field) theory, there is no big-bang type singularity as in the steady state theory of Bondi and Gold [9]. Narlikar [10] has pointed out that a proper consideration of matter creation can resolve the problem of singularity. Vishwakarma and Narlikar [11] have emphasized that the creation of matter plays a very crucial role in cosmology and provides a natural explanation to the various explosive phenomena occurring in local and extra galactic universe. Singh and Chaubey [12] have investigated Bianchi Type I, III, V, VI_0 and Kantowiki-Sachs universes in creation field cosmology where G is constant. Recently Bali and Tikekar [13] have investigated C-field cosmological model with variable G in flat FRW model i.e. for zero curvature.

In this paper, we have investigated C-field cosmological models with variable G in the frame work of FRW model for positive and negative curvature. To get the deterministic solution in terms of cosmic time t, we have assumed $G = R^n$ where $n = -1, -2$ and R is the scale factor. We find that creation field (C) increases with time and G, ρ (matter density) decreases with time in the model (21) while ρ is constant and G decreases with time in the model (41). The other physical aspects of the models are also discussed.

We consider the FRW model as

$$ds^2 = dt^2 - R^2(t)\left[\frac{dr^2}{1 - Kr^2} + r^2 d\theta^2 + r^2 \sin^2\theta \, d\phi^2\right] \tag{1}$$

where $K \neq 0$.

Einstein's field equation by introduction of C-field is modified as

$$R_i^j - \frac{1}{2} R g_i^j = -8\pi G[{}^m T_i^j + {}^C T_i^j] \tag{2}$$

where G (gravitational constant) is time dependent, ${}^m T_i^j$ the energy-momentum tensor for matter and ${}^C T_i^j$ the energy-momentum tensor for creation field. Following Hoyle and Narlikar [8], we take zero pressure in the matter field. Thus

$${}^m T_i^j = \rho v_i \, v^j \tag{3}$$

ρ is the homogeneous mass density, v^i the flow vector such that $v^1 = 0 = v^2 = v^3$, $v^4 = 1$. The energy-momentum tensor ${}^C T_i^j$ for creation field is given by

$${}^C T_i^j = -f\left[C_i C^j - \frac{1}{2} g_i^j C^\alpha C_\alpha\right] \tag{4}$$

where $f > 0$ and $C_i = \frac{dC}{dx^i}$. The field equation (2) with the help of (3) and (4) for the metric (1) leads to

$$\frac{3\dot{R}^2}{R^2} + \frac{3K}{R^2} = 8\pi G(t)\left[\rho - \frac{1}{2} f \dot{C}^2\right] \tag{5}$$

$$\frac{2\ddot{R}}{R} + \frac{\dot{R}^2}{R^2} + \frac{K}{R^2} = 4\pi G(t) f \dot{C}^2 \tag{6}$$

The conservation equation

$$[8\pi G T_i^j]_{;j} = 0$$

leads to

$$8\pi\dot{G}\left[\rho-\frac{1}{2}f\dot{C}^2\right]+8\pi G\left[\dot{\rho}-f\dot{C}\ddot{C}+3\rho\frac{\dot{R}}{R}-\frac{3\dot{R}}{R}f\dot{C}^2\right]=0 \tag{7}$$

which yields $\dot{C}=1$ when used in the source equation.

Equations (5) and (6) lead to

$$\frac{\ddot{R}}{R}+\frac{2\dot{R}^2}{R^2}+\frac{2K}{R^2}=4\pi G\rho \tag{8}$$

Using $\dot{C}=1$ in (6), we have

$$\frac{2\ddot{R}}{R}+\frac{\dot{R}^2}{R^2}+\frac{K}{R^2}=4\pi Gf \tag{9}$$

To obtain the deterministic solution of (9), we assume

$$G=R^n \tag{10}$$

where n is a constant and R is a function of t alone.

Equations (9) and (10) lead to

$$2\ddot{R}+\frac{\dot{R}^2}{R}+\frac{K}{R}=4\pi fR^{n+1} \tag{11}$$

Let us assume $\dot{R}=F(R)$

This leads to $\ddot{R}=FF'$ with $F'=\frac{dF}{dR}$. Thus (11) leads to

$$\frac{dF^2}{dR}+\frac{1}{R}F^2=4\pi fR^{n+1}-\frac{K}{R} \tag{12}$$

which leads to

$$F^2=\frac{4\pi fR^{n+2}}{n+3}-K \tag{13}$$

which again leads to

$$\frac{dR}{\sqrt{R^{n+2}-\frac{K(n+3)}{4\pi f}}}=\sqrt{\frac{4\pi f}{n+3}}dt \tag{14}$$

To obtain the determinate value of R in terms of cosmic time t, we consider two cases

Case I: $n=-1$

Equation (14) for $n=-1$ leads to

$$\frac{dR}{\sqrt{R-\frac{K}{2\pi f}}}=\sqrt{2\pi f}dt \tag{15}$$

From (15), we have

$$R=[At+B]^2+\frac{K}{2\pi f} \tag{16}$$

where

$$A = \frac{1}{2}\sqrt{2\pi f} \tag{17}$$

$$B = \frac{N}{2} \tag{18}$$

and N is the constant of integration. Thus we have

$$G = R^{-1} = \left[(At+B)^2 + \frac{K}{2\pi f}\right]^{-1} \tag{19}$$

From (8), (16) and (19), we have

$$8\pi\rho = \frac{[\frac{2A^2}{\pi f}+4]K + 20A^2[At+B]^2}{[(At+B)^2+\frac{K}{2\pi f}]} \tag{20}$$

Thus the metric (1) after using (16) leads to

$$ds^2 = dt^2 - \left[(At+B)^2 + \frac{K}{2\pi f}\right]^2 \left[\frac{dr^2}{1-Kr^2} + r^2\,d\theta^2 + r^2\sin^2\theta\,d\phi^2\right] \tag{21}$$

Now (7) leads to

$$8\pi[G\dot{\rho} + \dot{G}\rho] - 4\pi\dot{G}f\dot{C}^2 + 24\pi G\rho\frac{\dot{R}}{R} - 8\pi Gf\dot{C}\ddot{C} - 24\pi Gf\dot{C}^2\frac{\dot{R}}{R} = 0 \tag{22}$$

Substituting equations (16), (19) and (20) into equation (22), we have

$$\begin{aligned}\dot{C}^2\left[(At+B)^2 + \frac{K}{2\pi f}\right]^5 = \frac{1}{\pi f}\int\left[\frac{6K}{\pi f}A^2 + 2K + 20A^2(At+B)^2\right]\\ \times A(At+B)\left[(At+B)^2 + \frac{K}{2\pi f}\right]^3 dt\end{aligned} \tag{23}$$

To find the deterministic value of $\dot{C}$, we assume $A = 1$, $B = 0$. Thus (23) leads to

$$\dot{C}^2\left[t^2 + \frac{K}{2\pi f}\right]^5 = \frac{20}{\pi f}\left[\int\left[t^2 + \frac{K}{2\pi f}\right]^4 t\,dt\right] \tag{24}$$

we have

$$\dot{C}^2 = \frac{2}{\pi f} \tag{25}$$

then we have

$$\dot{C} = \sqrt{\frac{2}{\pi f}}$$

which leads to

$$C = \sqrt{\frac{2}{\pi f}}\,t \tag{26}$$

Taking $f=\frac{2}{\pi}>0$, we find $\dot{C}=1$ which agrees with the value used in the source equation. Thus creation field C is proportional to time t and the metric (1) for the constraints mentioned above, leading to

$$ds^2 = dt^2 - \left[t^2 + \frac{K}{2\pi f}\right]^2 \left[\frac{dr^2}{1-Kr^2} + r^2\,d\theta^2 + r^2\sin^2\theta\,d\phi^2\right] \tag{27}$$

The homogeneous mass density ρ, the Gravitational constant G, and the deceleration parameter q for the model (21) are given by

$$8\pi\rho = \frac{[\frac{2A^2}{\pi f}+4]K + 20A^2[At+B]^2}{[(At+B)^2 + \frac{K}{2\pi f}]} \tag{28}$$

$$G = \frac{1}{[(At+B)^2 + \frac{K}{2\pi f}]} \tag{29}$$

$$q = -\frac{[\frac{K}{\pi f}A^2 + 2A^2(At+B)^2]}{4A^2(At+B)^2} \tag{30}$$

$$C \propto t \tag{31}$$

Case (ii): $n=-2$

For $n=-2$, equation (14) leads to

$$\frac{dR}{\sqrt{1-\frac{K}{4\pi f}}} = \sqrt{4\pi f}\,dt \tag{32}$$

From equation (32), we have

$$R = [\sqrt{4\pi f - K}\,t + N] \tag{33}$$

where N is constant of integration. Thus we have

$$G = R^{-2} = [\sqrt{4\pi f - K}\,t + N]^{-2} \tag{34}$$

From (8), (33) and (34), we have

$$8\pi\rho = 16\pi f \tag{35}$$

Thus the metric (1) after using (33) leads to

$$ds^2 = dt^2 - (\sqrt{4\pi f - K}\,t + N)^2 \left[\frac{dr^2}{1-Kr^2} + r^2\,d\theta^2 + r^2\sin^2\theta\,d\phi^2\right] \tag{36}$$

Now substituting equations (33), (34) and (35) into (22), we have

$$\dot{C}^2[\sqrt{4\pi f - K}\,t + N]^4 = \int 4\sqrt{4\pi f - K}(\sqrt{4\pi f - K}\,t + N)^3\,dt \tag{37}$$

Equation (37) leads to

$$\dot{C}^2 = 1 \tag{38}$$

which leads to

$$\dot{C} = 1 \tag{39}$$

which leads to

$$C = t \tag{40}$$

Here we find $\dot{C} = 1$, which agrees with the value used in the source equation. Thus creation field C is proportional to time t and the metric (1) for the constraints mentioned above, leading to

$$ds^2 = dt^2 - [\sqrt{4\pi f - K}t + N]^2\left(\frac{dr^2}{1 - Kr^2} + r^2\, d\theta^2 + r^2 \sin^2\theta\, d\phi^2\right) \tag{41}$$

where $K = -1$.

The homogeneous mass density ρ, the gravitational constant G and deceleration parameter q for the model (36) are given by

$$\rho = 2f \tag{42}$$

$$G = [\sqrt{4\pi f - K}t + N]^{-2} \tag{43}$$

$$q = 0 \tag{44}$$

For the model (21), the matter density $\rho =$ constant for $K = 0, 1, -1$ when we take $A = 1$, $B = 0$, $G \to \infty$ when $t \to 0$, $G \to 0$ when $t \to \infty$ $|\frac{\dot{G}}{G}| \simeq \frac{1}{t} = H$. The deceleration parameter $(q) < 0$. Hence the model (21) represents an accelerating universe. Equation (31) shows that the creation field C increases with time. These match with the observations. For the model (41), the matter density $\rho = 2f =$ constant. Deceleration parameter $q = 0$. The metric (41) leads to Milne space–time where $K = -1$. Equations (26) and (40) show that C increases with time in both the models. The matter density is constant and C increases with time for the model (41). Referring to Hoyle and Narlikar [4], Hawking and Ellis [14], we may interpret this result as: The matter is supposed to move along the geodesic normal to the surface $t =$ constant. As the matter moves further apart, it is assumed that more matter is continuously created to maintain the matter density at constant value.

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